Enrollment No: ______________________ Exam Seat No:______________________

C.U.SHAH UNIVERSITY

Summer Examination-2017

Subject Name: Engineering Mathematics-I

Subject Code: 4TE01EMT2 **Branch: B.Tech(All)**

Semester: 1 **Date: 22/03/2017** **Time: 10:30 to 01:30** **Marks: 70**

Instructions:

(1) Use of Programmable calculator and any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1 Attempt the following questions: **(14)**

a) If $z=1+\sqrt{3}\,i$ then $|\bar{z}|=$ ______.

(a) 1 (b) 2 (c) $\sqrt{3}$ (d) $1-\sqrt{3}\,i$ s

b) Principal argument of $z=i+1$ is ______.

(a) $e^{\frac{3\pi}{4}i}$ (b) $\sqrt{2}$ (c) $e^{\frac{\pi}{4}i}$ (d) $e^{-\frac{\pi}{4}i}$

c) $e^{\frac{\pi}{2}i}=$ _______.

(a) 1 (b) -1 (c) i (d) $-i$

d) Find n[th] derivative of 3^x.

e) $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n=$ ______.

(a) 1 (b) e (c) $\log 1$ (d) $\frac{1}{e}$

f) $\lim_{x\to 0}\frac{x}{\tan 3x}=$ _____.

(a) 3 (b) $\frac{1}{3}$ (c) 1 (d) 0

g) n^{th} derivative of $y=\log(3-2x)$ is

(a) $\frac{2^n n!}{(3-2x)^{n+1}}$ (b) $\frac{(-1)^n(-2)^n n!}{(3-2x)^n}$ (c) $\frac{-(2)^n(n-1)!}{(3-2x)^n}$ (d) $\frac{(-1)^{n-1}(-2)^n(n-1)!}{(3-2x)^{n+1}}$

h) The series $x+\frac{x^3}{3!}+\frac{x^5}{5!}+....$represent expansion of

(a) $\sin x$ (b) $\cos x$ (c) $\sinh x$ (d) $\cosh x$

i) If $y=\cos^{-1}x$ then x equal to

(a) $1-\frac{y^2}{2!}+\frac{y^4}{4!}-...$ (b) $y-\frac{y^3}{3!}+\frac{y^5}{5!}-...$ (c) $1+y+\frac{y^2}{2!}+\frac{y^3}{3!}+...$ (d) none of these

j) State Euler's theorem for homogeneous function of two variables x and y.

k) Find $\frac{dy}{dx}$ if $x^2+y^2+1=0$

l) What is the value of $\frac{\partial}{\partial y}\left(y^x\right)=$ ______.

$(a)\,y^x$ $(b)\,y^x\log y$ $(c)\,xy^{x-1}$ $(d)\,x^y$

m) A square matrix A is called orthogonal if

(a) $AA^{-1}=I$ (b) $A^2=A$ (c) $AA^T=I$ (d) $A^2=I$

n) The rank of the non-zero scalar matrix of order 3 is ______.

(a) 1 (b) 2 (c) 3 (d) none of these

Attempt any four questions from Q-2 to Q-8

Q-2 Attempt all questions

a) Find the cube root of unity and prove that the sum of the roots is 0. **(05)**

b) Find the real and imaginary part of $i^{\ln(1+i)}$. **(05)**

c) Simplify: $\frac{(\cos 2\theta+i\sin 2\theta)^6(\cos\theta-i\sin\theta)^{15}}{(\cos 3\theta-i\sin 3\theta)^2(\cos 5\theta-i\sin 5\theta)^{-2}}$ **(04)**

Q-3 Attempt all questions

a) If $y=e^{m\cos^{-1}x}$then show that $\left(1-x^2\right)y_{n+2}-(2n+1)xy_{n+1}-\left(n^2+m^2\right)y_n=0$ **(05)**

b) Find the n[th] derivative of $y=\frac{x}{x^2+a^2}$. **(05)**

c) Expand $e^x\sin x$ in powers of x by Maclaurin's theorem up to the containing x^5 . **(04)**

Q-4 Attempt all questions

a) Evaluate: $\lim_{x\to 0}\left(\frac{1}{x^2}-\frac{1}{\tan^2 x}\right)$ **(05)**

b) Evaluate: $\lim_{x\to 0}\left(\frac{1}{x}\right)^{1-\cos x}$ **(05)**

c) Expand $f(x)=x^4-11x^3+43x^2-60x+14$ in powers of $(x-3)$. **(04)**

Q-5 Attempt all questions **(14)**

a) Solve the linear equation $\frac{1}{x}+\frac{1}{y}+\frac{2}{z}=8;\ -\frac{1}{x}-\frac{2}{y}+\frac{3}{z}=1;\ \frac{3}{x}-\frac{7}{y}+\frac{4}{z}=10$. **(05)**

b) Find the rank of the matrix $A=\begin{bmatrix}1 & 2 & 3 & -1\\ 2 & 1 & 3 & 1\\ 1 & 0 & 1 & 1\\ 0 & 1 & 1 & 1\end{bmatrix}$ by normal form. **(05)**

c) Evaluate: $\lim\limits_{x\to 0}\left(\frac{a^x+b^x+e^x-3}{2x}\right)$ **(04)**

Q-6 Attempt all questions

a) If $u=x^2\tan^{-1}\left(\frac{y}{x}\right)$ then find $\frac{\partial^2 u}{\partial x\partial y}$. **(05)**

b) If $x=r\cos\theta, y=r\sin\theta$ then prove that $\left(\frac{\partial r}{\partial x}\right)_y=\left(\frac{\partial x}{\partial r}\right)_\theta$ & $\left(\frac{\partial \theta}{\partial x}\right)_y=\frac{1}{r^2}\left(\frac{\partial x}{\partial \theta}\right)_r$. **(05)**

c) If $x=r\cos\theta, y=r\sin\theta$ then prove that $J\cdot J'=1$. **(04)**

Q-7 Attempt all questions

a) i) Define: Homogeneous function **(07)**

ii) Prove that $x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}=2\sin u\cos 3u$ if $u=\tan^{-1}\left(x^2+2y^2\right)$.

b) Find the Eigen values and Eigenvectors for the matrix $A=\begin{bmatrix}4 & 6 & 6\\ 1 & 3 & 2\\ -1 & -4 & -3\end{bmatrix}$. **(07)**

Q-8 Attempt all questions

a) Find the maxima and minima of $xy+27\left(\frac{1}{x}+\frac{1}{y}\right)$. **(05)**

b) Check the consistency and if consistent, solve the following system of equations **(05)**
$3x+3y+2z=1;\ x+2y=4;\ 10y+3z=-2;\ 2x-3y-z=5$

c) Find the inverse of the matrix $A=\begin{bmatrix}3 & -1 & 2\\ 4 & 1 & -1\\ 5 & 0 & 1\end{bmatrix}$ by Gauss Jordan method. **(04)**